Engineering Notes

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Laminar Velocity Profiles in Adverse Pressure Gradients

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Introduction

THE calculation of flow around and in the wake of hypersonic re-entry vehicles has renewed interest in the development of laminar boundary layers. Although many numerical techniques are available to compute the laminar boundary-layer development, it should be of value to have a simple technique to predict the shape of the velocity distribution at a given location. A recent study of subsonic laminar separation[1] has led to an accurate empirical representation of laminar velocity profiles approaching separation.

Analysis

The laminar velocity profile is represented in terms of the freestream pressure distribution and the Blasius flat plate boundary-layer function. The form of the equation is also determined so that the proper pressure gradient requirements at separation are met. The general velocity profile is assumed to have the form

$$u/U = A + Bf' + C(f')^n \tag{1}$$

in which A, B, C, and $n(n > 1)$ are functions of x only, f' is the Blasius solution of flat plates, and U is the velocity outside the boundary layer. The boundary conditions for the profile are

$$u = 0 \qquad (\partial u/\partial y)_w = \tau_w/\mu \qquad \text{at } y = 0$$
$$u = U \qquad \text{at } y \to \infty$$

where τ_w is the wall shear stress and ρ is the density of the fluid. The constant $A = 0$, and $B = 1 - C$. The relation

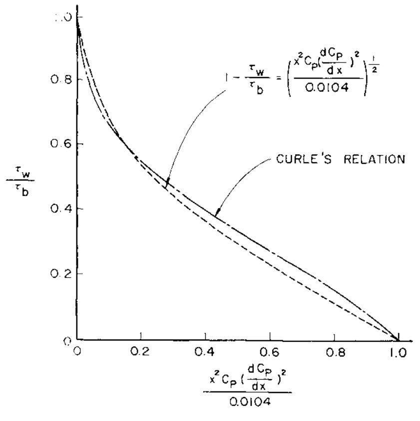

Fig. 1. Relation of τ_w/τ_b and $x^2C_p(dC_p/dx)^2$.

(1) must satisfy the exact Blasius zero pressure gradient solution; thus

$$B = \frac{\tau_w}{f''(0)\mu U(U/\nu x)^{1/2}}$$

where $\mu U(U/\nu x)^{1/2} f''(0) = \tau_b$, the flat plate wall shear stress. Equation (1) can now be written as

$$u/U = (\tau_w/\tau_b)f' + [1 - (\tau_w/\tau_b)](f')^n \tag{2}$$

The quantities (τ_w/τ_b) and n must now be expressed in terms of the external flow conditions. Curle[2] gives a relation between τ_w and τ_b in terms of the external pressure gradient.

$$x^2C_p(dC_p/dx)^2 = 0.0104[1 - (\tau_w/\tau_b)]^3[1 + 2.02(\tau_w/\tau_b)] \tag{3}$$

Equation (3) is plotted in Fig. 1 and can be employed in Eq. (2). However, due to the complex nature of Eq. (3), it is not convenient to use. An approximate relation

$$1 - \frac{\tau_w}{\tau_b} = \left[\frac{x^2C_p(dC_p/dx)^2}{0.0104}\right]^{1/2} \tag{4}$$

is also plotted on Fig. 1 and is employed with Eq. (2).

For the zero pressure flow the profile must reduce directly to the Blasius profile and for the separation case Eq. (2) becomes

$$u/U = (f')^n \qquad \text{for} \qquad \tau_w = 0 \tag{5}$$

Thus, n must have a value greater than one. Stratford[3] has demonstrated that the laminar velocity profile at separation depends upon C_{P_s}, where C_{P_s} is the pressure coefficient at separation found from Eq. (3) when $\tau_w = 0$. The power n was assumed to have the form

$$n = 1 + aC_{P_s}{}^b \tag{6}$$

The constants a and b were selected to give the best fit of Eq. (5) to the separation profiles given by Tani[4] for the flows $U_1 = 1 - X^m$ (where $m = 1,2,4$, and 8). Figure 2 compares the calculations from Eq. (5) for $a = 1.74$ and $b = 0.4$ with the separation profiles of Tani.

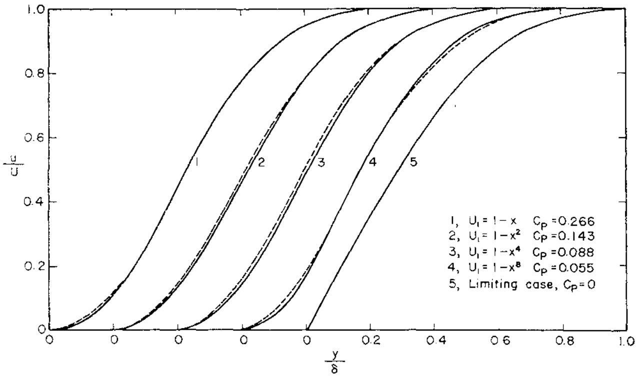

Fig. 2. Comparison of the laminar boundary-layer separation velocity profile with exact solutions.

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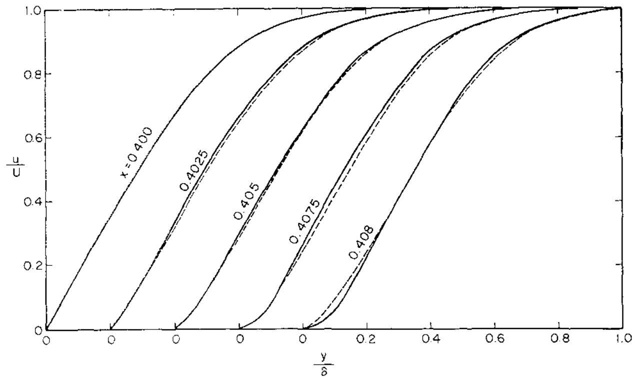

Fig. 3. Comparison of the approximate velocity profile with the computer solutions for the $U_1 = 1 - (x - 0.4)$.

The complete equation for the mean velocity distribution of laminar boundary layers can be written as

$$\frac{u}{U} = \left\{1 - \left[\frac{x^2 C_p \left(\frac{dC_p}{dx}\right)^2}{0.0104}\right]^{1/2}\right\} f' + \left[\frac{x^2 C_p \left(\frac{dC_p}{dx}\right)^2}{0.0104}\right]^{1/2} f'^{(1+1.74 C_{p_s}{}^{0.4})} \quad (7)$$

Equation (7) is compared with a family of velocity profiles obtained from a computer solution[1] of the flow $U_1 = 1 - (x - 0.4)$. This flow has a Blasius profile at $x = 0.400$ and is only solved for the region beyond $x = 0.400$. The boundary-layer thickness δ used in Figs. 2 and 3 was taken at $u/U = 0.995$ as a convenient method of presenting the comparison. The profile does not require a value of δ for general evaluation.

Although early work on separation viewed the velocity distribution as a unique profile, it is well-known that the actual shape varies with the external flow. Liu and Sandborn[5] have demonstrated that the velocity profile form factor at separation varies as a function of the pressure gradient. Thus, the variation of the velocity profile, given by Eq. (5) at separation, as a function of the pressure gradient is a necessary requirement.

References

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